rium the principal axes x_1, x_2, x_3 are taken, respectively, parallel to orbital angular velocity (and hence perpendicular to orbital plane), oppositely parallel to orbital linear velocity, and radial outward from gravitational center. Orbital angular perturbation thus enhances rotation $\dot{\alpha}$ about x_1, and if the orbital angular velocity is denoted by Ω, the total moment of momentum is evaluated as

$$\mathbf{h}_0 = \mathbf{i}_1(\Omega + \dot{\theta}' + \dot{\alpha})A + \mathbf{i}_2(\dot{\beta} - \Omega\gamma)B + \mathbf{i}_3(\dot{\gamma} + \Omega\beta)C \quad (6)$$

where principal axis unit vectors and moments of inertia A, B, C have been introduced. Note that Ω is the only quantity in parentheses which is not a small perturbation.

In order to evaluate the time derivative of $\mathbf{h}_0$, it is necessary to account for the rotation of coordinate axes and unit vectors resulting from three effects: basic orbital motion Ω, perturbation orbital motion θ', and local rotations having components $\dot{\alpha},\dot{\beta},\dot{\gamma}$. When this is done, one obtains

$$\begin{aligned} d\mathbf{h}_0/dt = {} & \mathbf{i}_1(\ddot{\alpha} + \ddot{\theta}')A + \\ & \mathbf{i}_2\{\ddot{\beta}B + \Omega^2(A - C)\beta + \Omega(A - B - C)\dot{\gamma}\} + \\ & \mathbf{i}_3\{\ddot{\gamma}C + \Omega^2(A - B)\gamma - \Omega(A - B - C)\dot{\beta}\} \end{aligned} \quad (7)$$

showing that, within the present small-perturbation analysis, orbital perturbation θ' affects only the component of moment of momentum which relates to motion parallel to the orbital plane. (The same is, of course, not true for large-disturbance motion.) When the right-hand side of Eq. (7) is equated to external torque moment, one has an extended form of Euler's rigid body equations, appropriate for motion about a point moving in space.

The gravitational torque moment $\mathbf{M}_0$ vanishes for the equilibrium orientation in space $\alpha = \beta = \gamma = 0$ (indeed, this is the condition that defines equilibrium under the action of gravity gradient forces), and its value has been found for small departures from this orientation to be[2]

$$\mathbf{M}_0 = -3\Omega^2(B - C)\alpha\mathbf{i}_1 - 3\Omega^2(A - C)\beta\mathbf{i}_2 \quad (8)$$

It is evident from the form of (8) that the moment opposes the displacement (static stability) when the two conditions are satisfied:

$$B - C > 0 \qquad A - C > 0 \quad (9)$$

These indicate the only limitations imposed upon the mass distribution and also show which space orientations 90° away from stable equilibrium must be unstable (by interchanging moments of inertia in pairs). The complete dynamic stability is determined by Eqs. (1) and (2), and the three scalar equations obtained by substituting (8) and (9) into (5) are

$$A\ddot{\alpha} + 3\Omega^2(B - C)\alpha = -A\ddot{\theta}' \quad (10)$$

$$B\ddot{\beta} + 4\Omega^2(A - C)\beta + \Omega(A - B - C)\dot{\gamma} = 0 \quad (11)$$

$$C\ddot{\gamma} + \Omega^2(A - B)\gamma - \Omega(A - B - C)\dot{\beta} = 0 \quad (12)$$

for a rigid satellite of arbitrary mass distribution.

Discussion of the Motion

The feature of greatest interest in the present problem is the fact that orbital perturbations affect only the α motions representing oscillations parallel to orbital plane. This occurs only through the term on the right-hand side of Eq. (10), where one may properly regard it as a forcing function for α motion, since the θ' disturbance is found from Eqs. (1) and (2) without regard for Eqs. (10–12). The natural frequency for this principal mode is seen at once to be

$$\omega_\alpha = \Omega 3^{1/2}[(B - C)/A]^{1/2} \quad (13)$$

In the case of greatest practical interest, with axial symmetry such that $A = B$, it is seen that ω_α is never greater than $3^{1/2}$ times the orbital frequency Ω. Since the forcing frequency given by Eq. (4) was seen to be equal to Ω for circular orbits, the natural α motion may be magnified appreciably by the nearness to resonance. Oscillations about x_2 and x_3 axes, given by β and γ, evidently are not affected, since these are coupled with each other but not with the α motion. It is easy to show that, for these symmetric configurations already described, β and γ motions are 90° out of phase with each other, and their frequency then is given by

$$\omega_\beta = \Omega \cdot 2 \cdot [1 - (3C/4A)]^{1/2} \quad (14)$$

Configurations of greatest inherent (static) stability correspond to mass concentrations close to x_3 axis (hence very small values of moment of inertia C); in this limit the frequency given by (14) still is only slightly greater than ω_α (the limiting values, in the ratio $3^{1/2}:2$, have been given correctly by Domojilova and her co-workers in the Russian literature[1]).

Conclusions

The characteristic rotational motions of gravity-gradient satellites do not affect the stability or the period of oscillations due to orbital disturbances, nor does the rotational motion induce an orbital perturbation. The converse is not true: orbital oscillations affect the rotational motion parallel to orbital plane, and the interaction is in the nature of an external forcing function that is sinusoidal. The closeness of forcing frequency to system natural frequency ω_α will require closer examination, particularly for eccentric orbits.

References

1 Domojilova, L. M., "Notes on orbital dynamics," Bull. Moscow Univ. III, no. 2, 82 (1961); in Russian.

2 Michelson, I., "Librational dynamic-response limits of gravity-gradient satellites," AIAA J. 1, 216 (1963).

Bow Shock Correlation for Slightly Blunted Cones

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When solving for the flow properties in the re-entry trail of a hypersonic nose cone, it is in general necessary first to solve for the detailed flow field in the vicinity of the body. However, in some cases (e.g., for relatively short bodies at high altitudes), and for the purposes of a parametric analysis, it is sufficient merely to specify the shape of the bow shock.[1,6] Thus, if a simple but accurate correlation can be used, based only on body geometry and freestream conditions, a considerable saving in expense and effort will result.

For highly blunted bodies (e.g., a hemisphere cylinder), the Van Hise correlation[2] based on the blast wave analogy yields good results. The equation for the shock shape in this case is

$$r/r_n = 1.424[C_D{}^{0.5}(x/r_n)]^{0.46} \quad (1)$$

where r and x are cylindrical polar coordinates, with x measured along the body axis, whereas r_n is the base radius of the body, and C_D is its drag coefficient. It has been found, however, that, for slightly blunted conical nose shapes of low drag coefficient, the Van Hise correlation is not satisfactory. For these cases, a modified correlation has been formulated.

Small angle conical nose shapes, capped with a spherical segment of radius r_T, with $r_T/r_n \ll 1$, will be considered.

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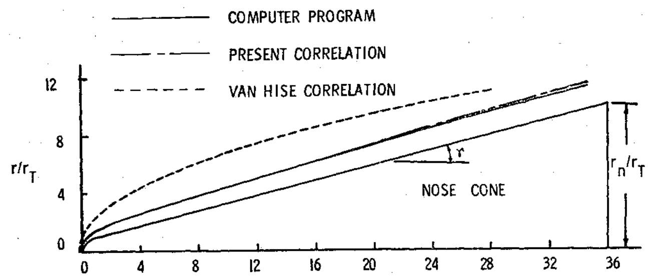

Fig. 1 Theoretical shock shape; $\gamma = 8.1139°$, $M_\infty = 12$, $r_T = 1$ in.

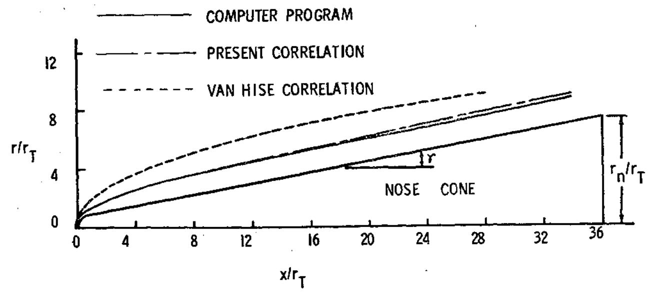

Fig. 2 Theoretical shock shape; $\gamma = 10.469°$, $M_\infty = 12$, $r_T = 1$ in.

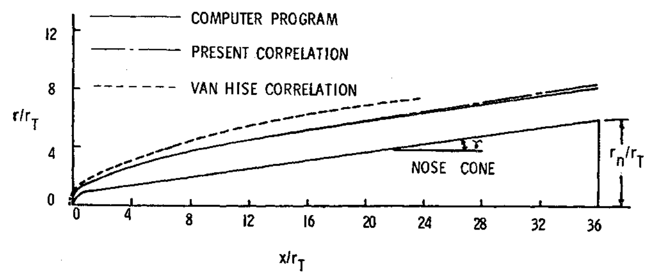

Fig. 3 Theoretical shock shape; $\gamma = 14.547°$, $M_\infty = 12$, $r_T = 1$ in.

Thus, the energy imparted to the flow in the vicinity of the stagnation point will be due primarily to the drag coefficient of the spherical segment C_{DT}. The shock shape in this region is, therefore, given by a form similar to the Van Hise correlation but based on the properties of the spherical cap:

$$r/r_T \cos\gamma = 1.424[C_{DT}^{0.5}(x/r_T \cos\gamma)]^{0.46} \qquad (2)$$

Here γ is the cone half angle. Modified Newtonian theory is used, so that

$$C_{DT} = 2 - \cos^2\gamma \qquad (3)$$

based on the reference area $\pi r_T^2 \cos^2\gamma$. Assuming now that the shock can be fared into the conical shock of angle δ corresponding to the freestream Mach number and the cone angle, Eq. (2) is used in the range

$$0 \leqslant r \leqslant r_T \cos\gamma\{0.984C_{DT}[(1/\sin^2\delta) - 1]\}^{0.426} \qquad (4)$$

For

$$r > r_T \cos\gamma\{0.984C_{DT}[(1/\sin^2\delta) - 1]\}^{0.426} \qquad (5)$$

the shock is extended as a straight line at angle δ. The angle δ can be obtained[7] from the equation

$$M_\infty \sin\delta = 4 + 1.01(M_\infty \sin\gamma - 3.43) \qquad (6)$$

for hypersonic flow.

The results of this correlation can be seen in the Figs. 1–5. In Figs. 1–3, comparison is made with the results of two Boeing numerical computer programs, one solving for the subsonic-transonic region by the method of Belotserkovskii,[3] and

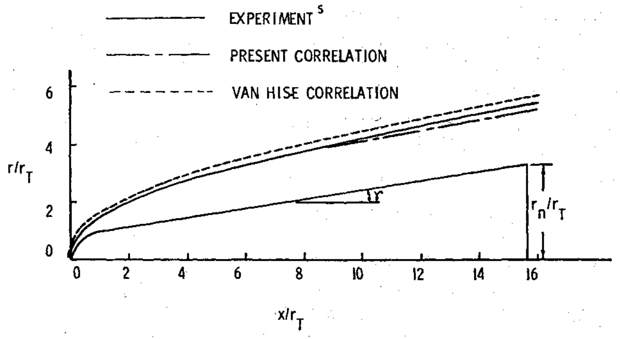

Fig. 4 Experimental shock shape; $\gamma = 9°$, $M_\infty = 18.9$, $r_T = 0.45$ in.

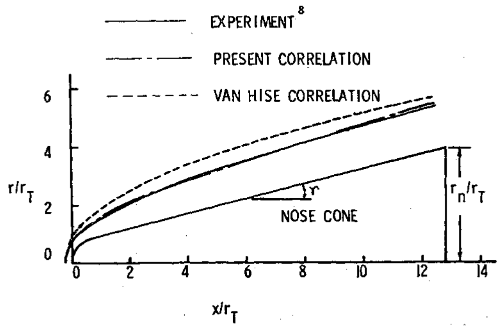

Fig. 5 Experimental shock shape; $\gamma = 14°$, $M_\infty = 6.1$, $r_T = 0.153$ in.

the other solving for the supersonic region by the method of characteristics.[4] Both programs are for real air in equilibrium. The Van Hise correlation also is included in the figures. Figures 4 and 5 present a comparison with experimental results.[5,8] The present correlation is virtually indistinguishable from both the computer program and the experiment up to at least 12 nose tip radii downstream. The Van Hise correlation compares most favorably in those cases when r_T/r_n is relatively large, so that the major contribution of nose cone drag comes from the spherical tip. For decreasing values of r_T/r_n, a larger percentage of the drag comes from the conical portion of nose cone, and it thus becomes essential to use the modified correlation presented here. It has been shown[1] that even the relatively small variation in shock shape in Fig. 4 can result in wake length variation on the order of 50%.

References

[1] Klaimon, J. H., "A parametric study of re-entry wakes," Boeing Co. Doc. 2-22040, Seattle, Wash. (October 1962).

[2] Van Hise, V., "Analytic study of induced pressure on long bodies of revolution with varying nose bluntness at hypersonic speeds," NASA TR-R-78 (1960).

[3] Kennet, H., "Hypersonic flow of high temperature air past blunt bodies," Boeing Co. Doc. D2-20592, Seattle, Wash. (July 1962).

[4] Eastman, D. W., "The formulation of the method of characteristics for two-dimensional or axial symmetric real gas flow with special emphasis on the method of machine solution," Boeing Co. Doc. D2-10597, Seattle, Wash. (December 1961).

[5] Lewis, C. H., "Pressure distribution and shock shape over slender cones at Mach numbers from 16 to 19," Von Kármán Facility, Aro Inc., AEDC-TN-61-81 (August 1961).

[6] Lykoudis, P. S., "Theory of ionized trails for bodies at hypersonic speeds," Rand Corp. RM-2682 (May 1961).

[7] Romig, M. F., "Conical flow parameters for air in dissociation equilibrium: final results," Convair Research Note 14, San Diego, Calif. (January 1958).

[8] Svarc, S. and Swartz, R., "An experimental investigation of the heat transfer distribution over a series of ballistic shapes at Mach 6.1," Boeing Co. Doc. D2-20561, Seattle, Wash. (to be released).